Math 55 Section 101 Quiz 9

Problem 1 Find **recurrence relations** and **initial conditions** that define the following sequences.

1.A (3 pt) $a_n = \binom{n}{3}$ for $n \geq 3$.

Solution: We can use the recursion $a_n = \frac{n!}{3!}(n-3)! = \frac{n}{n-3}\binom{n-1}{3} = \frac{n}{n-3}a_{n-1}$. Initial conditions are $a_3 = 1$.

1.B (3 pt) $a_n =$"The number of sequences of 2's and 5's that adds to n" with $n \geq 0$. For instance, $(2, 5, 2)$ is a sequence of 2's and 5's that adds to $n = 9$. Hint: You need 5 initial conditions.

Solution: We can use the recursion $a_n = a_{n-2} + a_{n-5}$. The initial conditions are just $a_0 = a_2 = 1$ and $a_1 = a_3 = a_4 = 0$.

Problem 2 (3 pt) Consider the sequence a_n with generating function $f(x) = \sum_{n=0}^{\infty} a_n x^n$, and suppose that a_n satisfies the recurrence relation $a_n = 4a_{n-2} + 1$ with initial conditions $a_0 = a_1 = 0$. Write down the corresponding equation satisfied by f. (I'm looking for an equation involving f, like $f(x) = x^5 f(x) + x^2$ for example).

Solution: The functional equation is $f(x) = 4x^2 f(x) + \frac{1}{1-x} - 1 - x$. Solving we get $f(x) = \frac{x^2}{(1-x)(1-2x)(1+2x)}$.

Problem 3 (1 pt, Challenge Problem) Consider the sequence a_n with generating function $f(x) = \sum_{n=0}^{\infty} a_n x^n$, and suppose that a_n satisfies the recurrence relation $a_n = -\frac{\omega^2}{n(n-1)}a_{n-2}$. Here ω is just some real number. Assume arbitrary initial conditions (use $a_0 = a$ and $a_1 = b$ if you want). Write down the general closed form solution for f.

Solution: The functional equation here ends up bring $\omega^2 f(x) = f''(x)$ (with some contributions from the initial conditions). We can solve this generally with $f(x) = A\sin(\omega x) + B\cos(\omega x)$. A and B depend on the initial conditions, but since we assumed that the initial conditions are arbitrary they can be anything (so we won't write them in term of the given a_0 and a_1).