

Math 55 Section 101 Quiz 5

Problem 1 (4 pt) Let T be the set of polynomials with even integer coefficients and even powers of x . Examples of such polynomials are $2x^4 + 6x^2$ or $16x^{32} - 2x^2 + 2$. $2x^3 + 4x^2$ is not allowed because x^3 is an odd power of x . $3x^2 + 2$ is not allowed because x^2 has an odd coefficient. **Give a recursive definition for T .**

Problem 2 The Fibonacci sequence is defined recursively as $f_0 = 0$, $f_1 = 1$ and $f_{i+2} = f_{i+1} + f_i$. Using strong induction, prove the following formula:

$$f_n = \frac{1}{\sqrt{5}} \left(\left(\frac{1 + \sqrt{5}}{2} \right)^n - \left(\frac{1 - \sqrt{5}}{2} \right)^n \right)$$

2.A (1 pt) What is “ $P(n)$ ” in this case?

2.B (2 pt) Prove the base case. (Caution: There should be 2 “base cases” in this situation.)

2.C (2 pt) Do the induction step (on the back please). Hint: Notice that $(\frac{1+\sqrt{5}}{2})^2 = 1 + \frac{1+\sqrt{5}}{2}$.