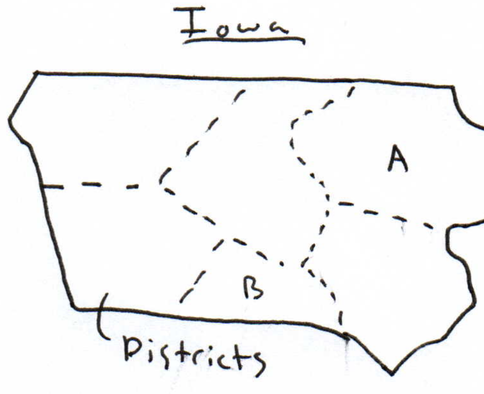
Math 55 Section 101, M 2/1/16 : Sets & Functions

Q] What event of great political significance is today?

A] Iowa Caucus!



Consider following electoral situations:

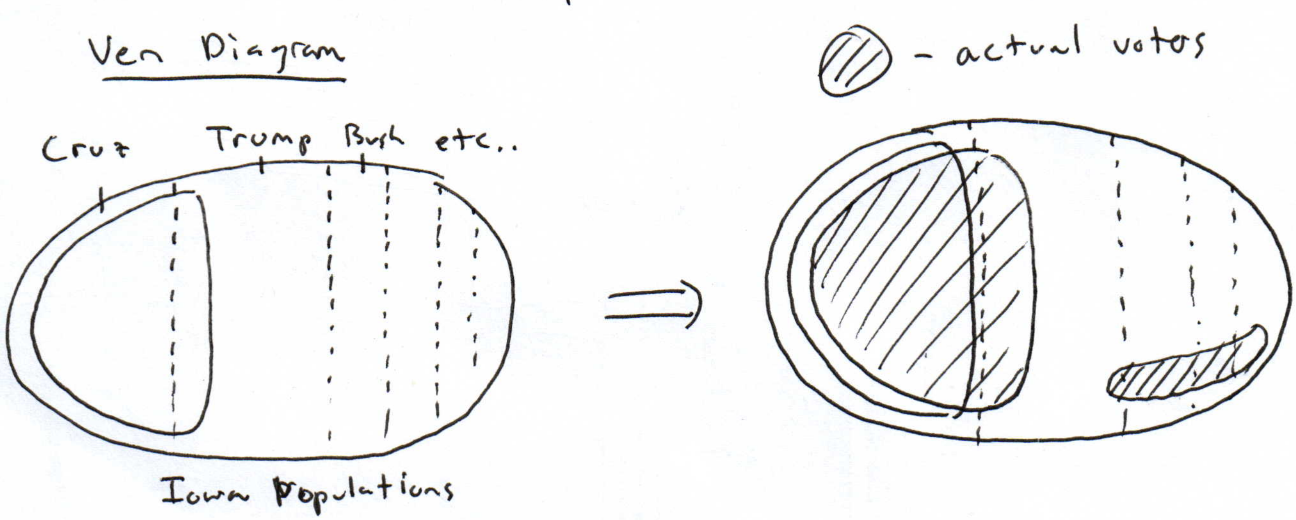
A - Heavily evangelical (christian/religious) district.

B - Rural district, all farms, farmowners or farm workers.

A] Some hypothetical "facts":

- Ted Cruz is campaigning entirely to religious voters
- Evangelical voters thus overwhelmingly support Cruz
- Bush, + Rubio campaign to moderates/non-religious conservatives.
- Evangelicals are 30% of residents in district A
- But! Evangelicals vote 90% of time, secular conservatives don't generally vote.
- Who wins? Obviously Cruz.

Ven Diagram



Maybe not so bad for non-Cruz candidates...

uh-oh.

Venn diagrams are helpful...

B | More "Facts"

- All adults in district B work in agriculture.
- Farm workers support Sanders (yay healthcare).
- Farm owners support Hillary (more pro-business).
- Who wins the district?
- Probably Sanders. Why?

- Every worker has a boss (Farm owner).
- Every owner probably employs >1 worker.

$\Rightarrow$ More workers than owners.

Translation:

$F :=$ set of Farm workers, $\mathcal{O} :=$ set of owners

$w : F \to \mathcal{O}$. Function w sends Farm~~er~~ worker to owner of his workplace. Map is surjective, not injective

$\Longrightarrow$ size of $F \gneq$ size of $\mathcal{O}$ 

Knowing about injection/surjection/bijection between sets gives info about relative size. (For $\underbrace{\text{finite}}_{\text{Also sort-of for } \infty \text{ sets.}}$ sets).

More mathy example:

A partition ~~is~~ of an integer (~~non-negative~~ positive) is an expression of n as a sum of other ~~non-negative~~ positive integers. Order doesn't matter.

Examples:

(a) $10 = \overbrace{5+3+1+1}^{\text{partition into 4 \#'s}}$ (same as $1+5+3+1$ or $3+1+1+5$)

(b) $25 = 10+5+5+2+\underbrace{1+1+1}_{\text{repeats are allowed}}$

Question:

How many ways to partition n into $\le k$ #'s each $\le j$?

Example:

(a) $10 = 3+3+2+2$ is partition of 10 into ≤ 4 #'s, each ≤ 10 (b/c $3, 2 \le 10$)

(b) $11 = 2+2+2+2+1+1+1$ is partition of 11 into ≤ 10 #'s, each ≤ 2.

Actually hard formula, not even sure if there is a general one...

BUT we can show:

Thm: # of ways to partition n into $\le k$ #'s each $\le j$

$\parallel$ equals

# of ways to partition n into $\le j$ #'s each $\le k$

$\Rightarrow$ can switch k with j and get same number.

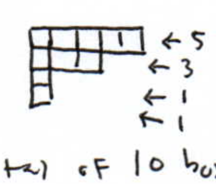
Do this with clever bijection!

Proof of Thm

$S :=$ set of ways to partition n into $\le k$ #'s each $\le j$

$T :=$ set of ways to partition n into $\le j$ #'s each $\le k$.

Can express partition in S as "table of boxes":

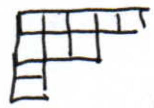
$10 = 5+3+1+1 \Rightarrow$ 

$\Longrightarrow$ Can do with any n boxes to get partition.

Partitions of n into $\leq k$ #'s each $\leq j$ have tables like so:

$\leq j$ in each row

From S $\Rightarrow$ $\leq k$ rows



n boxes

What is correspondence?

A: Flip!

Partitions of n into $\leq j$ #s each $\leq k$ have tables like so:

From T $\Rightarrow$

$\leq k$

$\leq j$

Flip

$\leq j$

$\leq k$

$\Leftarrow$ in S!

There is a "Flip" function $F: S \to T$ giving bijection.

$\Rightarrow$ size of S equals size of T! $\square$

Lesson: Bijections tell you #'s / sizes of sets are equal <u>even when you don't know actual sizes!</u> Cool