Quiz 1: 8.1, 8.3

Problem 1 Find the following indefinite integral.

$$\int x^2 e^{8x} dx$$

int. by parts #1

$u = x^2 \qquad dv = e^{8x} dx$

$du = 2x\, dx \qquad v = \frac{1}{8} e^{8x} dx$

integration by parts #2

$u = \frac{1}{4} x \qquad dv = e^{8x}$

$du = \frac{1}{4} \qquad v = \frac{1}{8} e^{8x}$

$$\int x^2 e^{8x} dx = \frac{x^2}{8} e^{8x} - \int \frac{2x}{8} e^{8x} dx = \frac{x^2}{8} e^{8x} - \frac{1}{32} x e^{8x} + \int \frac{1}{32} e^{8x} dx$$

$$\boxed{= \left(\frac{x^2}{8} - \frac{x}{32} + \frac{1}{256}\right) e^{8x} + C}$$

Problem 2 Compute the present value and accumulated money flow after a 10 year period at 4 percent interest, where the flow rate at 1000's of dollars per year is

$$f(t) = 20e^{-.01t}$$

$P = \int_0^T f(t) e^{-rt}$, $A = e^{rT} \cdot P$

$T = 10$, $r = .04$, $f(t) = 20e^{-.01t}$

$.05 = \frac{1}{20}$

$$P = \int_0^{10} 20 e^{-.01t} \cdot e^{-.04t} dt = \int_0^{10} 20 e^{-.05t} dt$$

$$= \frac{20}{-1/20} \cdot e^{-.05t} \Big|_0^{10} = \boxed{400(1 - e^{-1/2})}$$

$$A = e^{.04 \cdot 10} \cdot P = \boxed{400(1 - e^{-1/2}) \cdot e^{2/5}}$$

Problem 3 Find the following integral

$$\int_0^{\pi/2} \cos(\theta)\sin(\theta)d\theta$$

u - substitution $u = \sin\theta$ (or $\cos\theta$)

$du = \cos\theta \, d\theta$ (or $-\sin\theta \, d\theta$)

$$\int_0^{\pi/2} \cos\theta \sin\theta \, d\theta = \int_0^1 u \, du = \frac{1}{2}u^2\Big|_0^1 = \boxed{\frac{1}{2}}$$

Problem 4 Find the following integral

This one is tricky.

$$\int \cos(x)e^x dx$$

2x int. by parts $\Rightarrow$ (*)

$$\int \cos(x)e^x dx = \cos(x)e^x + \int + \sin(x)e^x \, dx \quad \} \text{ I.B.P \#1}$$

$u = \cos(x)$ $dv = e^x dx$

$du = -\sin(x)$ $v = e^x$

$$(*) \int \sin(x) e^x = \sin(x)e^x - \int \cos(x) e^x \, dx \quad \} \text{ I.B.P \#2}$$

$u = \sin(x)$ $dv = e^x dx$

$du = \cos(x)$ $v = e^x$

$$\Rightarrow \int \cos(x) e^x \, dx = (\cos(x) + \sin(x)) e^x - \int \cos(x) e^x dx$$

$$\Rightarrow 2 \cdot \int \cos(x) e^x dx = (\cos(x) + \sin(x)) e^x$$

$$\Rightarrow \int \cos(x) e^x dx = (\cos(x) + \sin(x)) e^x + C$$

C technically.