Math 535A: Differential Topology (3 Units)

Course Description. This is a graduate level course on differential topology. The course will cover basic manifold theory (manifolds, vector bundles, transversality, isotopies, tensors, differential forms and Lie groups); basic differential topology (Morse theory and handlebody theory); and basic symplectic topology.

Prerequisites. Students should have a strong background in real analysis (Math 425a), topology (Math 440), linear algebra (Math 235) and calculus (Math 226). Key background concepts include topological spaces, continuity, quotient spaces, fundamental group, multivariable differentiation and integration. Background in algebraic topology (Math 540) is also helpful.

Course Goals. This course aims to provide sufficient mastery of important research tools in the differential topology. By the end of the the first part, the student should obtain

- competency with basic concepts in differential topology: manifolds, vector bundles, embeddings, submersions, isotopies, transversality, tensors, differential forms and Lie groups.
- competency with coordinate-free computations and calculus on manifolds: Lie derivatives, exterior derivatives, interior product, integration, Lie brackets, pullback.
- a good list of elementary examples of manifolds, vector bundles and other objects encountered in the course (e.g. spheres, tori, projective spaces, etc).

Instructor. Julian Chaidez (`julian.chaidez@usc.edu`).

Assistant/Grader. Tianle Liu (`tianleli@usc.edu`).

Lecture Information. MWF 10-10:50 in KAP 141.

Office Hours Information. MW 11-12 in KAP 263 (Math Center).

Course Text. This class will not follow a book but the following books are useful references. Rough guidance by section will be given in the lecture schedule below.

- John M. Lee. *Introduction To Smooth Manifolds.* Springer. 2nd Edition.
- John Milnor. *Morse Theory.* Princeton University Press (1963).

Course Grades. At the end of the semester, grades will be computed by the following formula.

$$\text{Grade} \quad = \quad .1 \times \text{Scribing} \quad + \quad .1 \times \text{Exercises} \quad + \quad .2 \times \text{Quizzes} + .6 \times \text{Exams}$$

Each student will receive a grade update after each midterm via email. This will consist of a summary of your current grade and a short paragraph discussing your performance so far.

Scribing. This course will seek to generate a set of notes for use in future versions. This will be structured as follows.

- **Selection.** A scribe will be selected at random at the start of each class. Once you are selected, you won't be selected again until everyone else has scribed.
- **Overleaf.** The main course notes will be kept in an Overleaf file. After class, you will be required to typset your notes into the Overleaf file.

- **Due Date.** The scribed notes are required to be in the Overleaf file **within one week** of the lecture that you scribed.
- **Grading Policy.** You will get full credit for fully scribed, neat lecture notes. An unexcused absence when you are scribe will result in a scribe grade of zero for that cycle.
- **Revisions.** I will ask you to revise your scribed lecture if it's not well written, sufficiently complete or formatted well. Please come to office hours if you need help or have questions!

Quizzes. A 10 minute quiz will be administered at the start of class every Friday, consisting of a single question written on the board. Please bring a pen or pencil!

Exercises. A small number of exercises will be assigned at every lecture.

- **Due Day.** Solutions to **four exercises** are due on Fridays in class.
- **Formatting.** Handwritten or LaTeX solutions are both fine.
- **Collaboration Policy.** Students may collaborate on their solutions.
- **Grading Policy.** Solutions will only be graded for completion and effort.

Exams. There will be three in-class midterms and a final exam.

- **Midterm Exams.** The midterms will be a 45 minute, in class exam with 6 questions (3 true/false, 2 examples and a proof).
- **Missed Midterm.** If you will be absent for a midterm, that midterm will be dropped from your overall exam grade. Contact the instructor if you anticipate more missed exams.
- **Final Exam.** The final exam will be a 2 hour exam at the scheduled final exam time corresponding to our course time. The format will be 6 true/false, 4 examples, 3 proofs.

Statement For Students With Disabilities. Any student requesting academic accommodations based on a disability is required to register with Disability Services and Programs (DSP) each semester. A letter of verification for approved accommodations can be obtained from DSP. Please be sure the letter is delivered to me (or to TA) as early in the semester as possible. DSP is located in GFS 120 and is open 8:30 a.m.-5:00 p.m., Monday through Friday. Website for DSP (`https://dsp.usc.edu/`) and contact information: (213) 740-0776 (Phone), (213) 740-6948 (TDD only), (213) 740-8216 (FAX) `dspfrontdesk@usc.edu`.

Statement On Academic Integrity. USC seeks to maintain an optimal learning environment. General principles of academic honesty include the concept of respect for the intellectual property of others, the expectation that individual work will be submitted unless otherwise allowed by an instructor, and the obligations both to protect one's own academic work from misuse by others as well as to avoid using another's work as one's own. All students are expected to understand and abide by these principles. SCampus, the Student Guidebook, contains the University Student Conduct Code (see University Governance, Section 11.00), while the recommended sanctions in Appendix A.

Emergency Preparedness/Course Continuity In A Crisis. In case of a declared emergency if travel to campus is not feasible, USC executive leadership will announce an electronic way for instructors to teach students in their residence halls or homes using a combination of Blackboard, teleconferencing, and other technologies. See the university's site on Campus Safety and Emergency Preparedness.

Lecture Schedule

Date	Topics	References
	Introduction	
M 1/12	course overview, goals and logistics, background	
	Smooth Maps In $\mathbb{R}^n$	
W 1/14	smooth maps in $\mathbb{R}^n$, inverse function and constant rank thms	Lee §C
F 1/16	partitions of unity, smooth extension and approximation	Lee §2
M 1/19	**Martin Luther King Day**	
	Manifolds	
W 1/21	smooth manifolds, smooth maps, diffeomorphisms	Lee §1
F 1/23	constructions (product, quotients and fibers), examples	Lee §1
	Vector Bundles	
M 1/26	vector bundles, bundle maps, sections, pullback, sub-bundles	Lee §10
W 1/28	operations on vector bundles	
F 1/30	(co)tangent bundle, (co)vector fields	Lee §3,11
	Special Maps, Rank And Transversality	
M 2/2	immersions/submersions/embeddings, normal/vertical bundles	Lee §4
W 2/4	transversality, examples and non-examples, Sard's theorem	Lee §6
F 2/6	parametric and generic transversality, (weak) Whitney embedding	Lee §6
M 2/9	**Midterm I**	
	Isotopies And Flows	
W 2/11	isotopies, flows, flow generated by vector fields, isotopy extension	Lee §9
F 2/13	Lie bracket of vector fields, Lie derivatives	Lee §8
M 2/16	**President's Day**	
	Tensors And Differential Forms	
W 2/18	tensor products, alternating algebra	Lee §12
F 2/20	tensors, differential forms	Lee §12
M 2/23	exterior derivative, Cartan formula	Lee §14
W 2/25	determinant bundle, orientation bundle, orientations	Lee §15
F 2/27	integration, Stokes theorem	
	Lie Groups	
M 3/2	Lie groups, homomorphisms, sub-groups, examples	Lee §7
W 3/4	Lie algebras, Lie bracket,	Lee §7
F 3/6	Lie group actions	Lee §7

Date	Topics	References
	Riemannian Manifolds	
M 3/9	bundle metrics, Riemannian metrics, isometries, existence	Lee §13
W 3/11	Riemannian distance, examples	Lee §13
F 3/13	Levi-Civita connection	Lee §13
	Spring Break	
M 3/23	**Midterm II**	
	Algebraic Topology Primer	
W 3/25	homotopy groups, intersection pairing	
F 3/27	de Rham cohomology, properties, Betti numbers	
	Morse Functions And Handlebody Theory	
M 3/30	Morse functions, index, existence, Morse lemma	Milnor
W 4/1	handles, attachment data	Milnor
F 4/3	handle creation/cancellation, handle slides, Cerf's theorem	
M 4/6	2-manifolds: diagrams and classification of surfaces	
W 4/8	3/4-manifolds: Heegaard/Kirby diagrams, examples	
F 4/10	**Midterm III**	
	Distributions And Foliations	
M 4/13	distributions, involutivity, foliations, examples	Lee §19
W 4/15	Frobenius theorem	Lee §19
F 4/17	slice theorem, quotient manifold theorem	Lee §21
	Symplectic Topology	
M 4/20	symplectic manifolds, isotropics/coisotropics/Lagrangians, examples	Lee §22
W 4/22	Moser trick, Darboux theorem	Lee §22
F 4/24	current research and open problems	Lee §22
	Contact Topology	
M 4/27	contact manifolds, Legendrians, examples	
W 4/29	Grey stability, Liouville and Weinstein manifolds	
F 5/1	current research and open problems	
	Final Exam	