Math 535B: Differential Geometry (3 Units)

Course Description. This is an advanced graduate course on differential geometry and geometric analysis. Topics covered include Riemannian manifolds, connections, curvature, geodesics, minimal surfaces, symplectic manifolds, (almost-)complex manifolds and Kahler manifolds.

Prerequisites. Students should have a strong background in real analysis (525a), differential topology (535a) and algebraic topology (540). The basics of partial differential equations (555a) and functional analysis will also be helpful.

Course Goals. This course aims to provide sufficient mastery of important research tools. By the end of the the course, the student should obtain

- a solid understanding of the basics of differential geometry (Riemannian manifolds, principle bundles, connections, curvature, calculus of variations).
- a good grasp of contemporary tools in the area (minimal surfac theory, Ricci flow, etc).

Instructor. Julian Chaidez (`julian.chaidez@usc.edu`).

Lecture Information. MWF 10-11 in KAP 427.

Office Hours Information. MWF 11-12 in KAP Math Center.

Course Website. The course website is `julianchaidez.net/2025f_math535b.html`. Thanks to the small class size, we will not need Brightspace for this class.

Textbook And Readings. The course will follow several textbooks and notes available online. See the lecture and reading schedule below.

Course Grades. At the end of the semester, grades will be computed by the following formula.

$$\text{Grade} \;=\; .4 \times \text{Problem Sets} \;+\; .3 \times \text{Oral Midterm} + \;.3 \times \text{Oral Final}$$

Problem Sets. Four problem sets consisting of textbook exercises will be assigned as follows.

- **Due Day.** Solutions are due on Fridays in class, following the schedule below.
- **Formatting.** Homework should be handwritten.
- **Collaboration Policy.** Students may collaborate on their solutions.
- **Late Policy.** Every student will be allowed two problem set extensions by one week.

Oral Exams. Two oral exams will be administered during the semester, an oral midterm and an oral final. The format will be the following.

- **Schedule.** The midterm and final will be scheduled for **Th 10/30** and **Tu 12/2** respectively. Every student will individually set up a time to take it.
- **Format.** Each exam will be short (45 minutes). I will pose three questions related to the lectures, readings and/or homeworks, and you will explain your thoughts at a board.
- **Preparation.** A detailed list of materials and theorems that you should be familiar with will be provided before each exam.

Statement For Students With Disabilities. Any student requesting academic accommodations based on a disability is required to register with Disability Services and Programs (DSP) each semester. A letter of verification for approved accommodations can be obtained from DSP. Please be sure the letter is delivered to me (or to TA) as early in the semester as possible. DSP is located in GFS 120 and is open 8:30 a.m.-5:00 p.m., Monday through Friday. Website for DSP (`https://dsp.usc.edu/`) and contact information: (213) 740-0776 (Phone), (213) 740-6948 (TDD only), (213) 740-8216 (FAX) `dspfrontdesk@usc.edu`.

Statement On Academic Integrity. USC seeks to maintain an optimal learning environment. General principles of academic honesty include the concept of respect for the intellectual property of others, the expectation that individual work will be submitted unless otherwise allowed by an instructor, and the obligations both to protect one's own academic work from misuse by others as well as to avoid using another's work as one's own. All students are expected to understand and abide by these principles. SCampus, the Student Guidebook, contains the University Student Conduct Code (see University Governance, Section 11.00), while the recommended sanctions are located in Appendix A.

Emergency Preparedness/Course Continuity In A Crisis. In case of a declared emergency if travel to campus is not feasible, USC executive leadership will announce an electronic way for instructors to teach students in their residence halls or homes using a combination of Blackboard, teleconferencing, and other technologies. See the university's site on Campus Safety and Emergency Preparedness.

Lecture Schedule (Part I)

Date	Topics	Reading
	Introduction And Riemannian Manifolds	
W 9/3	the three geometries, geometric functionals, calculus of variations	
F 9/5	Riemannian manifolds, isometries, existence, distance	G+ 52-58
M 9/8	volume,ju sub-manifolds, products, examples	G+ 58-65
	Connections And Curvature	
W 9/10	principal bundles, associated bundles, examples	
F 9/12	affine connections, principal bundle connections, Levi-Civita	G+ 70-75
M 9/15	parallel transport, curvature of connections	G+ 75-80
W 9/17	Riemannian, Ricci and scalar curvatures, examples	G+ 129-135
	Geodesics	
F 9/19	geodesics, local existence/uniqueness	G+ 80-89
M 9/22	exponential map, completeness, Hopf-Rinow	G+ 96-100
W 9/24	first and second variation, Jacobi vector fields, applications	G+ 141-148
F 9/26	geodesic flow, dynamics, applications	G+ 109-115
	Minimal Surfaces	
M 9/29	area functional, variations, maximum principle	M 1-6
W 10/1	calibration, second variational formula	M 6-12
F 10/3	monotonicity, Bernstein Theorem, stability condition	M 12-20
M 10/6	stability condition	M 20-29
W 10/8	Simon's equation, Schoen-Simons-Yau Theorem	M 29-33
M 10/13	Schoen-Simons-Yau Theorem, current developments	M 33-38
	Metrics Of Positive Curvature	
W 10/15	Einstein metrics, Einstein-Hilbert functional, variations, Ricci flow	B 1-5
F 10/17	short time existence, de Turck trick, uniqueness of Ricci flow	B 5-12
M 10/20	evolution of length, distance. Uhlenbeck trick.	B 12-17
W 10/22	evolution of curvatures, scalar maximum principle	B 17-24
F 10/24	applications of maximum principle, viscocity solutions	B 24-30
M 10/27	vector bundle maximum principle	B 30-37
W 10/29	applications, Hamilton's Ricci positive theorem	B 37-41
Th 10/30	**Oral Midterm**	

Lecture Schedule (Part II)

Date	Topics	Reading
	Systolic Geometry	
F 10/31	systoles, Pu's inequality, Gromov's systolic inequality	
M 11/3	Guth's proof for the n-torus, current developments	G
	General Relativity	
W 11/5	Semi-Riemanniann (and Lorentzian) metrics, Einstein equations	
F 11/7	examples, Kerr metric, FLRW metrics	
	Symplectic Manifolds	
$\star$	Symplectic manifolds, symplectomorphisms, Lagrangians, Hamiltonians	
$\star$	Darboux theorem, Weinstein neighborhood theorem, contact manifolds	
	Complex And Kahler Manifolds	
M 11/10	(Almost)-complex manifolds, holomorphic maps, examples	
W 11/12	Dolbeault complex, Newlander-Nirenberg theorem	
F 11/14	Kahler manifolds, isometries, Kahler potential, examples	SW 1-8
W 11/19	complex and holomorphic line bundles, Chern class, ampleness	SW 13-19
Tu 12/2	**Oral Final**	
	Calabi Yau Metrics	
F 11/21	Kahler-Ricci flow, parabolic Schwarz lemma, 3rd order estimate	SW 19-27
M 12/1	curvature estimates, Monge-Ampere	SW 27-34
W 12/3	estimates for volume, normalized flow	SW 34-41
F 12/5	Calabi-Yau theorem, Yau's zeroeth order estimate, bootstrap	SW 41-48

Reading List

- **[B]** *Lecture notes on Ricci flow.* Richard Bamler (scribed by Ao Sun). `drive.google.com/file/d/1fcUOu2ceWFzIiwWJe4Aw83H0PBeDm8Br/view`
- **[G]** *Systolic inequalities and minimal surfaces.* Larry Guth. `arxiv.org/abs/0903.5299`.
- **[G+]** *Riemannian Geometry.* Syvestre Gallot, Dominique Hulin, Jacques Lafontaine. Springer-Verlag.
- **[M]** *Surfaces minimales : theorie variationelle et applications.* Fernando Coda Marques. `web.math.princeton.edu/~rcabral/pdfs/minimalsurfaces.pdf`.
- **[SW]** *Lecture notes on the Kähler-Ricci flow.* Jian Song and Ben Weinkove. `arXiv:1212.3653`.

Homework Schedule

HW	Problems	Due Date
1	TBD	F 9/19
2	TBD	F 10/17
3	TBD	F 11/7
4	TBD	F 12/5