Math 540: Algebraic Topology (3 Units)

Course Description. This is a graduate level introduction to algebraic topology with a focus on homology and its applications. We will cover singular homology, Eilenberg-Steenrod axioms, basic homological algebra, homotopy groups, covering spaces and characteristic classes.

Prerequisites. Students should have a strong background in basic topology (Math 440), abstract algebra (Math 510b) and linear algebra (Math 235). Key background concepts include topological spaces, continuity, topological quotients, groups, rings, modules.

Course Goals. This course will aim to (a) provide sufficient mastery of algebraic topology to pass the screening exam in topology and (b) introduce important research tools in the subject. By the end of the course, the student should be able to

- apply the fundamental properties of homology and cohomology, e.g. the Eilenberg-Steenrod axioms and Mayer-Vietoris, to exercises and research problems.
- apply the fundamental properties of homotopy groups, e.g. Hurewicz, Van Kampen and the Puppe sequence, to exercises and research problems.
- apply the theory of covering spaces to exercises and research problems.
- apply the fundamental properties of the Steiffel-Whitney classes and Chern classes.

Instructor. Julian Chaidez (`julian.chaidez@usc.edu`).

Assistant/Grader. Tina Peng (`pengtina@usc.edu`).

Lecture Information. MWF 11-11:50 in KAP 414.

Office Hours Information. MF 4-5 in KAP 245 or KAP 263 (Math Center).

Course Website. The course website is `julianchaidez.net/2024f_math540.html`. Thanks to the small class size, we will not need Brightspace for this class.

Textbook. There will be two main textbooks that we will use in this course.

- Joseph J. Rotman. *An Introduction To Algebraic Topology.* Springer Graduate Textbooks.
- John Milnor and James D. Stasheff. *Characteristic Classes.* Princeton University Press.

Course Grades. At the end of the semester, grades will be computed by the following formula.

$$\text{Grade} \quad = \quad .4 \times \text{Problem Sets} \quad + \quad .2 \times \text{Midterm} \quad + \quad .4 \times \text{Final}$$

Grade Updates. Each student will receive a pre-midterm, post-midterm, pre-final and post-final grade update via email. This will consist of a summary of your current grade and a short paragraph discussing your performance so far, and what you could do to improve. If you are struggling, I will ask to meet with you to discuss potential solutions in person.

Readings. Readings are assigned for every class meeting following the schedule below. At a minimum, you should spend 30 minutes skimming the reading before class. Lectures will not cover everything in the reading and you may need materials in the reading for the homework.

Problem Sets. Weekly problem sets consisting of textbook exercises will be assigned as follows.

- **Due Day.** Solutions are due on Wednesdays in class with one exception (see schedule below). Printed or written solutions must be provided in person, at the start of class.
- **Formatting.** Students are encouraged (but not required) to write solutions in LaTeX.
- **Collaboration Policy.** Students may collaborate on their solutions.
- **Late/Drop Policy.** No late problem sets will be accepted. The bottom two problem sets will be dropped from the students overall problem set grade. This allows any student to skip one or two problem sets if needed.
- **Grading Policy.** Solutions will be graded carefully for clarity, rigor and correctness.

Exams. The midterm and final will each consists of two equally weighted components.

- **Take-Home Component.** This part of the exam will be a long assignment consisting of original (non-book) problems. This will replace the homework for that week.
- **In-Class Component.** The midterm will be a 45 minute, in class exam with 6 questions (3 true/false, 2 examples and a proof). The final will be a 2 hour exam on the scheduled final day/time. Both exams will test all materials up to 1 week before the exam date.
- **Collaboration Policy.** Collaboration is permitted on the take-home components, with the inclusion of the names of partners, but every student must write original solutions.

Statement For Students With Disabilities. Any student requesting academic accommodations based on a disability is required to register with Disability Services and Programs (DSP) each semester. A letter of verification for approved accommodations can be obtained from DSP. Please be sure the letter is delivered to me (or to TA) as early in the semester as possible. DSP is located in GFS 120 and is open 8:30 a.m.-5:00 p.m., Monday through Friday. Website for DSP (`https://dsp.usc.edu/`) and contact information: (213) 740-0776 (Phone), (213) 740-6948 (TDD only), (213) 740-8216 (FAX) `dspfrontdesk@usc.edu`.

Statement On Academic Integrity. USC seeks to maintain an optimal learning environment. General principles of academic honesty include the concept of respect for the intellectual property of others, the expectation that individual work will be submitted unless otherwise allowed by an instructor, and the obligations both to protect one's own academic work from misuse by others as well as to avoid using another's work as one's own. All students are expected to understand and abide by these principles. SCampus, the Student Guidebook, contains the University Student Conduct Code (see University Governance, Section 11.00), while the recommended sanctions are located in Appendix A.

Emergency Preparedness/Course Continuity In A Crisis. In case of a declared emergency if travel to campus is not feasible, USC executive leadership will announce an electronic way for instructors to teach students in their residence halls or homes using a combination of Blackboard, teleconferencing, and other technologies. See the university's site on Campus Safety and Emergency Preparedness.

Lecture Schedule (Part I)

Date	Topics	Reading
	Introduction And Background (Ch 0,1,2)	
M 8/26	overview, fixed point theorem, categories and functor	Ch 0, p. 1-13
W 8/28	homotopy, convexity, contractibility, cones, path connectedness	Ch 1, p. 14-30
F 8/30	simplicies, affine spaces, affine maps	Ch 2, p. 31-39
M 9/2	**Labor Day**	
	Fundamental Group And Singular Homology (Ch 3,4)	
W 9/4	fundamental groupoid, the fundamental group functor	Ch 3, p. 39-56
F 9/6	singular complex, singular homology functor	Ch 4, p. 57-68
M 9/9	dimension axiom, homotopy axiom, Hurwicz (statement)	Ch 4, p. 68-80
	Homological Algebra (Ch 5)	
W 9/11	complexes, chain maps, chain homotopies	Ch 5, p. 86-93
F 9/13	exact sequences, relative homology	Ch 5, p. 93-105
	Excision And Mayer-Vietoris (Ch 6)	
M 9/16	excision, Mayer-Vietoris, start excision proof	Ch 6, p. 105-119
W 9/18	finish excision proof, applications of excision	Ch 6, p. 111-130
	Simplicial And Cellular Homology (Ch 7,8)	
F 9/20	(abstract) simplicial complexes, simplicial approximation	Ch 7, p. 131-142
M 9/23	simplicial homology,comparison with singular, calculations	Ch 7, p. 142-164
W 9/25	cell attachments, CW complexes	Ch 8, p. 180-212
F 9/27	cellular homology	Ch 8, p. 212-227
	Homology Axioms (Ch 9)	
M 9/30	natural transformations, Eilenberg-Steenrod axioms	Ch 9, p. 228-237
W 10/2	tensor products, universal coefficients	Ch 9, p. 253-265
F 10/4	Eilenberg-Zilber theorem, Kunneth formula	Ch 9, p. 265-272
	Properties Of Fundamental Group (Ch 4,7)	
M 10/7	Hurewicz map, Hurewicz theorem	Ch 4, p. 80-85
W 10/9	Van Kampen, computations	Ch 7, p. 164-180
	Fall Recess	
M 10/14	**In-Class Midterm**	

Lecture Schedule (Part II)

Date	Topics	Reading
	Covering Spaces (Ch 10)	
W 10/16	covering spaces, basic properties	Ch 10, p. 272-284
F 10/18	covering transformations	Ch 10, 284-295
M 10/21	existence, orbit	Ch 10, 295-311
	(Higher) Homotopy Groups (Ch 11)	
W 10/23	function spaces, (co)group objects	Ch 11, p. 312-323
F 10/25	loop space, suspension	Ch 11, p. 323-334
M 10/28	higher homotopy groups	Ch 11, p. 334-344
W 10/30	exact sequence of a pair	Ch 11, p. 344-355
F 11/1	exact sequence of a fibration	Ch 11, p. 355-368
M 11/4	**No Lecture**	Ch 11, p. 368-377
	Cohomology (Ch 12, MS §1,A)	
W 11/6	cohomological algebra, singular cohomology, properties	Ch 12, p. 377-390
F 11/8	graded algebras, cup product, cap product	Ch 12, p. 390-402
	Veterans Day	
W 11/13	topological manifolds, fundamental class	MS §A, p. 270-275
F 11/15	compactly supported cohomology, Poincare duality	MS §A, p. 275-280
	Vector Bundles And Characteristic Classes (MS)	
M 11/18	smooth manifolds, examples, constructions	MS §1, p. 3-13
W 11/20	vector bundles, examples, constructions	MS §2,3 p. 12-37
F 11/22	Stieffel-Whitney classes, applications	MS §4, p. 37-55
M 11/25	oriented bundles, Euler class	MS §9, p. 95-105
	Thanksgiving Break (Tested Material Ends Here)	
M 12/2	the Thom isomorphism theorem	MS §10, p. 105-115
W 12/4	complex vector bundles, complex manifolds	MS §13, p. 149-155
F 12/6	Chern classes, properties, computations	MS §14, p. 155-173
W 12/11	**Final Exam**	

Homework/Takehome Schedule

HW	Problems	Due Date
1	0.3, 0.4, 0.6, 0.12, 0.15, 0.18, 1.1, 1.2, 1.5	W 9/4
2	2.4, 3.8, 3.19, 3.21, 3.22, 4.2, 4.3	W 9/11
3	4.4, 4.6, 4.10, 4.11, 4.13, 5.1, 5.2, 5.3, 5.4, 5.7, 5.8	W 9/18
4	5.11, 5.13, 5.15, 5.20, 5.21, 6.4, 6.5, 6.21, 6.23	W 9/25
5	7.2, 7.3, 7.10, 7.19, 7.20, 7.26, 8.2, 8.17, 8.36	W 10/2
	Takehome Midterm Component	M 10/14
6	9.48, 4.13, 4.14, 7.47, 10.1, 10.3, 10.4	W 10/23
7	10.15, 10.16, 10.18, 10.19, 10.33, 11.8, 11.9	W 10/30
8	11.12, 11.24, 11.25, 11.29, 11.31, 11.32	W 11/6
9	12.1, 12.2, 12.3, 12.15, 12.17, 12.18, 12.19	W 11/13
10	MS A-2, 1-A, 1-B, 2-A, 2-B, 3-B, 3-E, 4-B, 4-C	M 11/25
	Takehome Final Component	F 12/6