Math 55 Section 101 Quiz 8

Note You can leave your final answers in unsimplified form (no need to break out a calculator).

Problem 1 You and your friend have 4 coins that you want to divide between the two of you. You both decide to play a game: you will flip the coins. If there are an even number of heads, your friend get the coins and if there are an odd number you get the coins.

1.A (3 pt) First suppose the coins are fair: they all have a 1/2 chance of flipping heads. Who has a better chance of winning?

Solution: You both have an equal chance of winning. There are $\binom{4}{1}+\binom{4}{3}=8$ ways of getting an odd number of heads, and 16 total possible outcomes, so the probability of you winning is 50 percent.

1.B (3 pt) Now suppose that the coins are *not* fair: they all have a 1/10 chance of being heads. Who has a better chance of winning?

Solution: Your friend has a better chance. The probability of flipping k heads where the coins have probability q of being heads is $q^k(1-q)^{4-k}\binom{4}{k}$. To get an odd number of heads, $k=1$ or $k=3$. Thus the probability (for $q=.1$) is:

$$(1/10)^1(1-1/10)^3\binom{4}{1}+(1/10)^3(1-1/10)^1\binom{4}{3}=\frac{4}{10^4}(9^3+9)=\frac{2952}{10^4}\simeq 30 \text{ percent}$$

Problem 2 (4 pt) This seasons flu is going around, and you've caught it. You go to your doctor and ask if you'll be sick for more than a week. He says that about 50 percent of flu patients are sick for more than a week. However, he also observes that you have developed a fever; 80 percent of his patients that are sick for more than a week develop a fever, while only 20 percent of those who are sick for less than a week develop one. Given that you have a fever, what are your chances of being sick for more than a week? (Answer on the back).

Solution: This is a Bayes' theorem problem. If F is the event of developing a fever and W is the probability of having the flu for more than a week, then we want $p(W|F)$ and we have $p(F|W), p(F|\bar{W}), p(W)$ and $p(\not{W})$. Namely, $p(F|W)$ is the probability of getting a fever for people who have the flu for more than a week, which is .8 according to the doctor. $p(F|\bar{W})$ is the probability of getting a fever for people who've had the flue for less than a week, which is .2, also according to your doctor. $p(W)$ and $p(\bar{W})$ (the probability of having the flue for more than a week and the opposite) are .5 and .5 respectively. Thus by Bayes:

$$p(W|F)=\frac{p(F|W)p(W)}{p(F|W)p(W)+p(F|\bar{W})p(\bar{W})}=\frac{.8\cdot .5}{.8\cdot .5+.2\cdot .5}=.8=80 \text{ percent}$$