Math 55 Section 101 Quiz 2

Problem 1 (4.1 Q 31) Find each of these values.

1.A $(-133 \mod 23 + 261 \mod 23) \mod 23$
Answer: $(-133 \mod 23 + 261 \mod 23) \mod 23 = (-133 + 261) \mod 23 = 128 \mod 23 = 13 \mod 23$

1.B $(457 \mod 23 \cdot 182 \mod 23) \mod 23$
Answer: $(457 \mod 23 \cdot 182 \mod 23) \mod 23 = -3 \mod 23 \cdot 21 \mod 23 = -3 \cdot 21 \mod 23 = -63 \mod 23 = 6 \mod 23$

Problem 2 (4.2 Q 6) Convert the octal expansion of each of these integers to a binary expansion. **How to do this:** Use the method that I discussed in section!

2.A $(572)_8$ **Answer:** 101111010

2.B $(1604)_8$ **Answer:** 001110000010

2.C $(423)_8$ **Answer:** 100010011

2.D $(2417)_8$ **Answer:** 010100001111

Problem 3 (1 pt) (4.1 Q 17) Show that if n and k are positive integers, then $\lceil n/k \rceil = \lfloor (n-1)/k \rfloor + 1$.
Answer: Let $n = ak + b$ with b between 0 and k. Then:

$$\lceil n/k \rceil = a + \lceil b/k \rceil = a + 1 \text{ and } \lfloor (n-1)/k \rfloor + 1 = \lfloor (ak+b)/k \rfloor + 1 = a + \lfloor \frac{b-1}{k} \rfloor + 1 = a + 1$$

It's important to notice here that the range of b is important, since if it were outside of $\{1, \ldots, k-1\}$, b/k would not have floor 0 and ceiling 1.

Problem 4 (1 pt) (4.2 Q 32) Show that a positive integer is divisible by 7 if and only if the sum of its octal digits is divisible by 7. **Answer:** Let $n = \sum_{i=0}^{k} a_i 8^i$, so the octal expansion is $(a_k a_{k-1} \ldots a_1 a_0)_8$. Then we see that:

$$n \mod 7 = (\sum_i a_i 8^i) \mod 7 = (\sum_i (a_i \mod 7)(8^i \mod 7)) \mod 7$$

Now observe that $8 \mod 7 = 1$ because $8 = 7 + 1$. Thus:

$$8^i \mod 7 = (8 \mod 7)^i \mod 7 = 1^i \mod 7 = 1 \mod 7$$

So we can write the sum from before as so:

$$n \mod 7 = (\sum_i (a_i \mod 7)(8^i \mod 7)) \mod 7 = (\sum_i (a_i \mod 7)(1 \mod 7)) \mod 7$$

$$= \sum_i (a_i \mod 7) \mod 7 = (\sum_i a_i) \mod 7$$

In particular, $n = 0 \mod 7$ if and only if $\sum_i a_i = 0 \mod 7$. Thus n is divisible by 7 if and only if $\sum_i a_i$ is.