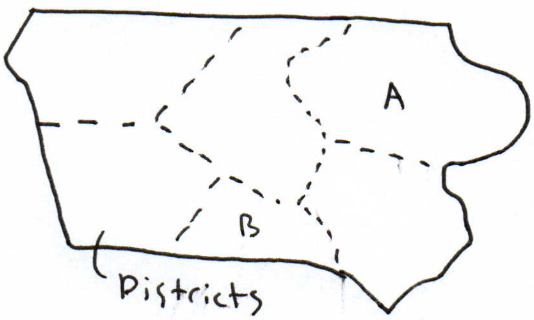
Math55 Section 101, M 2/1/16 : Sets & Functions

Q] What event of great political significance is today?

A] Iowa Caucus!

Iowa



Consider following electoral situations:

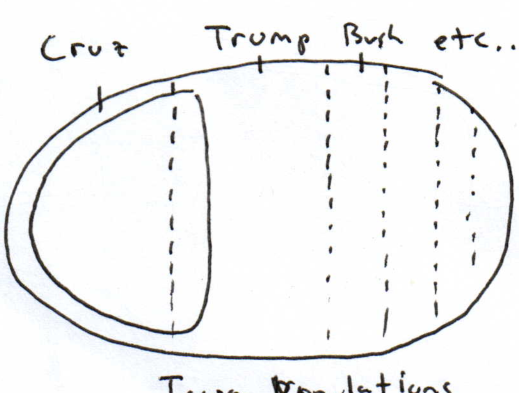
A - ~~E~~ Heavily evangelical (christian/religious) district.

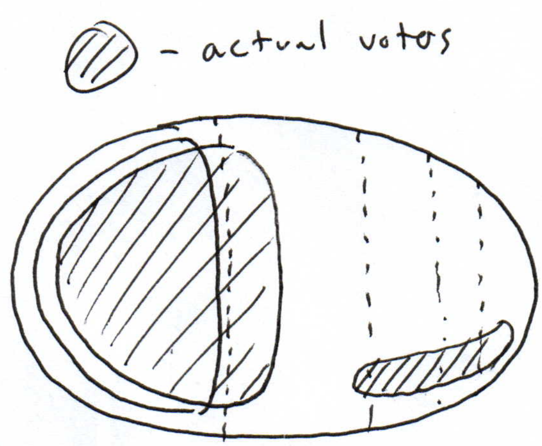
B - Rural district, all farms, farmowners or farm workers.

A] Some hypothetical "facts":

- Ted Cruz is campaigning entirely to religious voters
- Evangelical voters thus overwhelmingly support Cruz
- Bush, + Rubio campaign to moderates/non-religious conservatives.
- Evangelicals are 30% of residents in district A
- But! Evangelicals vote 90% of time, secular conservatives don't generally vote.
- Who wins? Obviously Cruz.

Ven Diagram





Maybe not so bad for non-Cruz candidates...

uh-oh.

Venn diagrams ~~are~~ are helpful...

B | More "Facts"

- All adults in district B work in agriculture.
- Farm workers support Sanders (yay healthcare).
- Farm owners support Hillary (more pro-business).
- Who wins the district?
- Probably Sanders. Why?

- Every worker has a boss (Farm owner).
- Every ~~[illegible]~~ owner probably employs >1 worker.

$\Rightarrow$ More workers than owners.

Translation:

$F :=$ set of Farm workers, $\mathcal{O} :=$ set of owners

$w: F \to \mathcal{O}$. Function w sends Farm worker to owner of his workplace. Map is surjective, not injective

$\Longrightarrow$ size of $F \underset{\neq}{\geq}$ size of $\mathcal{O}$

Knowing about injection/surjection/bijection between sets gives info about relative size. (For $\underbrace{\text{finite}}_{}$ sets).

Also sort-of for ∞ sets.

More mathy example:

A partition of n is of an integer (~~non-negative~~ positive) is an expression of n as a sum of other ~~non-negative~~ positive integers. Order doesn't matter.

Examples:

(a) $10 = \overbrace{5+3+1+1}^{\text{partition into 4 \#'s}}$ (same as $1+5+3+1$ or $3+1+1+5$)

(b) $25 = 10+5+5+2+\underbrace{1+1+1}_{\text{repeats are allowed}}$

Question:

How many ways to partition n into $\leq k$ #'s each $\leq j$?

Example:

(a) $10 = 3+3+2+2$ is partition of 10 into ≤ 4 #'s, each ≤ 10 (b/c $3, 2 \leq 10$)

(b) $11 = 2+2+2+2+1+1+1$ is partition of 11 into ≤ 10 #'s, each ≤ 2.

Actually hard formula, not even sure if there is a general one...

BUT we can show:

Thm: # of ways to partition n into $\leq k$ #'s each $\leq j$

$\parallel$ equals

# of ways to partition n into $\leq j$ #'s each $\leq k$

$\Rightarrow$ can switch k with j and get same number.

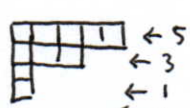
Do this with clever bijection!

Proof of Thm

$S :=$ set of ways to partition n into $\leq k$ #'s each $\leq j$

$T :=$ set of ways to partition n into $\leq j$ #'s each $\leq k$.

Can express partition in S as "table of boxes":

$10 = 5+3+1+1 \Rightarrow$ 

total of 10 boxes

$\Longrightarrow$ Can do with any n boxes to get partition.