Student Learning Guide

This is a learning guide for PhD students who are interested in working with me. If you're interested, we can set up a reading course on one of these topics.

Foundations. The following topics are required foundations for PhD students who want to work with me. Ideally you would learn this material by the start of Year 2 of PhD.

- **Algebraic Topology.** Read and do exercises in Rotman [17], or get an A in Math 540.
- **Manifold Topology.** Read and do exercises in Spivak [19], or get an A in Math 535a.
- **Symplectic Geometry.** Read and do exercises in Ch 1-4 of McDuff-Salamon [14].
- **Morse Theory.** Read Milnor Pt I [15]. This book is an old classic.
- **Floer Theory.** Read and do exercises in McDuff-Salamon Ch 11 [14] and Salamon [18].

The following additional topics are useful but not strictly necessary.

- **Examples/Constructions.** Read McDuff-Salamon Ch 6-8 [14] and Geiges Ch 4 [7].
- **Complex (Algebraic) Geometry.** Read Griffiths-Harris Ch 0-1 [9] (another classic).
- **Functional Analysis.** Read Shakarchi-Stein [20] or get an A in Math 525b.

Research Areas. Here is a big list of my research interests with recommended readings.

- **Symplectic Field Theory.** The theory of holomorphic curves in symplectic cobordisms, and the corresponding Floer theory. Read Wendl [22] for an indepth introduction.
- **Embedded Contact Homology.** A version of SFT for contact 3-manifolds with powerful applications. Read Hutchings [10] for an indepth introduction.
- **Symplectic Homology.** A version of Hamiltonian Floer theory with many applications. Read Wendl [21] to start, then Abouzaid [1] for a comprehensive introduction.
- **Fukaya Categories.** A categorical enhancement of Floer theory that plays a central role in mirror symmetry. Read Auroux [3] to start, then maybe Ganatra-Pardon-Shende [6].
- **Seiberg-Witten Theory.** An gauge theory invariant with powerful applications to 3/4-manifold topology. Read Morgan [16] for the basics and then Mrowka-Kronheimer [13].
- **Quantitative Symplectic Geometry.** This is the study of capacities and spectral invariants from Floer theory. Read McDuff-Salamon Ch 12 [14].
- **Contact 3-Manifolds.** The study of contact topology in low dimensions. Read Geiges Ch 3-4 [7] and Etnyre's notes [4].
- **Smooth 4-Manifolds.** Methods in 4-manifold topology. Read Gompf-Stipcicz [8].
- **Systolic Geometry.** The study of inequalities relating minimal widths, sizes and volumes in Riemannian manifolds and their generalizations. Read some of Katz [12].
- **Smooth Dynamics.** The study of smooth maps and flows. Read some of Katok-Hasselblatt [11] for a broad introduction, then Fisher-Hasselblatt [5].
- **Billiard Dynamics.** The study of the motion of billiard balls - an old subject with many open problems and connections to many fields. Read Athreya-Masur [2].

Open Problems. Math research is about questions and open problems. To give you a better idea of what my research is about, here is a list of some of my favorite open problems. I am happy to tell you more about any of these!

Conjecture 1 (Schoenflies). *Any smooth embedding of the 3-sphere $\iota : S^3 \to \mathbb{R}^4$ extends to a smooth embedding of the 4-ball $\iota : D^4 \to \mathbb{R}^4$.*

Conjecture 2 (Symplectic-Hyperbolic). *There is no closed symplectic 4-manifold that admits a hyperbolic metric.*

Conjecture 3 (Salamon). *The space of symplectic forms on $\mathbb{C}P^2$ is connected.*

Conjecture 4 (Fake Weinstein Manifolds). *There exists a Liouville manifold (X, λ) that is diffeomorphic but not symplectomorphic to a Weinstein manifold.*

Conjecture 5 (Exotic Fillings). *There is a pair of exact fillings W and X of the same contact 3-manifold Y that are diffeomorphic but not symplectomorphic.*

Conjecture 6 (Symplectic Smale Conjecture). *Let (X, ω) be a closed symplectic 4-manifold*

$$X = \mathbb{CH}^2/\Gamma$$

where $\mathbb{CH}^2$ is complex-hyperbolic 2-space and Γ acts by Kahler isometries. Then the identity component of the symplectomorphism group is contractible.

Conjecture 7 (Weinstein Conjecture). *Every closed contact manifold (Y, ξ) with contact form α has a closed Reeb orbit.*

Conjecture 8 (Closing Property). *The Reeb flow on a closed contact manifold (Y, ξ) with contact form α satisfies the smooth closing lemma in the sense of Smale.*

Conjecture 9 (Billiards Orbits). *Every triangular billiard table $T \subset \mathbb{R}^2$ has a closed orbit.*

Conjecture 10 (Billiard Complexity). *The billiard complexity $N(T, k)$ of a convex polygonal billiard table T, defined as the count*

$$N(T, k) = \#\{\gamma \,:\, \gamma \text{ is a billiard trajectory with } \leq k \text{ bounces}\}$$

grows like k^2 as $k \to \infty$ for a generic billiard table.

Conjecture 11 (Symmetric Viterbo). *Let $X \subset \mathbb{R}^{2n}$ be a centrally symmetric convex domain and let c_{EHZ} denote the EHZ capacity. Then*

$$\frac{c_{EHZ}(X)^n}{\text{vol}(X)} \leq \frac{c_{EHZ}(B^{2n})^n}{\text{vol}(B^{2n})}$$

Conjecture 12 (Simple Type). *Let X be a simply connected closed oriented 4-manifold with spin-c structure $\mathfrak{s}$. Then the Seiberg-Witten invariants satisfy*

$$SW(X, \mathfrak{s}) = 0 \qquad \text{if the virtual dimension } d(X, \mathfrak{s}) \text{ is non-zero}$$

Conjecture 13 (Algebraic Weinstein, Contact Homology Version). *There are no closed contact manifolds (Y, ξ) with contact homology algebra $CH(Y, \xi) = \mathbb{Q}$.*

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